

Direct Shear Box Assessment of Sand–Rubber Mixtures: Strength, Ductility, and a Performance Index

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Article Info

Received : 03.10.2025

Accepted : 16.12.2025

DOI: 10.21605/cukurovaumfd.1796691

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Keywords

Sand–rubber mixture

Shear strength

Performance index

Sustainable geotechnics

Waste tires

How to cite: TUNA, Ş.Ç., (2025). Direct Shear Box Assessment of Sand–Rubber Mixtures: Strength, Ductility, and a Performance Index. Çukurova University, Journal of the Faculty of Engineering, 40(4), 977-991.

ABSTRACT

The reuse of waste materials in geotechnical engineering promotes sustainability while enhancing soil performance. This study investigates the effect of shredded waste rubber on sand through consolidated drained direct shear tests at two relative densities ($D_r \approx 30\%$ and 70%) with 0–10% rubber content. Rubber inclusion reduced brittleness and improved ductility, with mixtures containing 2.5–5% achieving the most favorable strength–deformation balance. In loose sand, cohesion and friction angle increased up to 5%, while in dense sand cohesion rose slightly but friction angle declined at higher contents.

A new Performance Index (PI) is proposed in this study, integrating energy absorption, ductility, and residual-to-peak strength into a unified framework. The PI confirmed the 2.5–5% range as optimum, providing a practical tool for mixture evaluation. These findings indicate that low rubber contents are suitable for seismic embankments, retaining walls, and transportation subgrades, while higher contents may be more effective in lightweight and energy-dissipative applications.

Kum–Lastik Karışımlarının Doğrudan Kesme Kutusu ile Değerlendirilmesi: Mukavemet, Süneklik ve Performans İndeksi

Makale Bilgileri

Geliş : 03.10.2025

Kabul : 16.12.2025

DOI: 10.21605/cukurovaumfd.1796691

Sorumlu Yazar

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Anahtar Kelimeler

Kum–lastik karışımı

Kesme mukavemeti

Performans indeksi

Sürdürülebilir geoteknik

Atık lastikler

Atf şekli: TUNA, Ş.Ç., (2025). Kum–Lastik Karışımlarının Doğrudan Kesme Kutusu ile Değerlendirilmesi: Mukavemet, Süneklik ve Performans İndeksi. Çukurova Üniversitesi, Mühendislik Fakültesi Dergisi, 40(4), 977-991.

ÖZ

Geoteknik mühendisliğinde atık malzemelerin yeniden kullanımı, sürdürülebilirliği teşvik ederken zemin performansını da artırmaktadır. Bu çalışma, parçalanmış atık lastiklerin kum üzerindeki etkisini konsolide-drenajlı doğrudan kesme deneyleri ile incelemektedir. İki farklı bağıl sıklıkta ($D_r \approx \%30$ ve $\%70$) ve $\%0$ – $\%10$ lastik katkı oranı ile yapılan deneylerde, lastik ilavesinin kırılma direnci azalttığı ve sünekliği artırdığı; özellikle $\%2.5$ – $\%5$ aralığındaki karışımların en uygun mukavemet–deformasyon dengesini sağladığı belirlenmiştir. Gevşek kumlarda kohezyon ve içsel sürtünme açısı $\%5$ 'e kadar artarken, sıkı kumlarda kohezyon hafif artış göstermiş, ancak sürtünme açısı yüksek katkı oranlarında azalmıştır. Bu çalışmada ayrıca enerji yutma, süneklik ve artık/tepe mukavemet oranını bütünleştiren yeni bir Performans İndeksi (PI) önerilmiştir. PI, $\%2.5$ – $\%5$ aralığının optimum olduğunu doğrulamış ve karışımların değerlendirilmesinde pratik bir araç sunmuştur. Bulgular, düşük lastik katkı oranlarının sismik dolgular, istinat duvarları ve ulaşım altyapısı alt dolguları için uygun olduğunu; daha yüksek oranların ise hafif ve enerji sönmeyici dolgular için daha etkili olabileceğini göstermektedir.

1. INTRODUCTION

The reuse of waste materials in geotechnical engineering has attracted growing interest over the past two decades due to the combined pressures of sustainable development, environmental protection, and cost efficiency. Among these materials, discarded automobile tires represent a major environmental challenge worldwide, as millions of tons are generated annually and their non-biodegradable nature leads to long-term hazards and uncontrolled accumulation in landfills. Recycling waste tires for geotechnical purposes has therefore emerged as a promising approach, simultaneously addressing sustainability and soil improvement objectives [1-2].

Granular soils, particularly sand, are widely used in earth structures such as embankments, foundations, and backfills because of their high permeability and ease of compaction. However, their inherent brittleness and low ductility under monotonic or cyclic loading present a key limitation, especially in seismic regions. Clean sands typically exhibit negligible cohesion and sudden strength loss after peak, with little energy absorption capacity [3]. To overcome these shortcomings, various reinforcing materials have been examined, including fibers, geosynthetics, and shredded waste rubber, with the aim of improving both shear strength and deformation capacity.

Recent studies on sustainable soil and material modification have highlighted that alternative additives, including biopolymers and waste tire rubber, primarily influence the balance between strength and ductility in geomaterials [4,5]. Previous research has shown that rubber inclusion can provide multiple benefits [6], through sixty direct shear tests, reported that ~5% rubber content at moderate density enhanced the internal friction angle, while higher contents or dense conditions reduced peak shear resistance, illustrating the dual role of rubber. Other studies confirmed that shredded tires reduce the unit weight of mixtures, increase ductility, and enhance energy absorption, delaying brittle failure [7,8]. [2] emphasized their effectiveness as lightweight backfill, while [9] and [3] highlighted the influence of rubber content and geometry on shear response. More recent investigations [10-13] further revealed that low rubber contents enhance resistance, but excessive amounts increase liquefaction susceptibility or reduce interparticle friction. Complementary numerical modeling studies have clarified the microscale mechanisms of contact force redistribution and deformation in rubber–sand systems [14].

Although these contributions highlight the potential of sand–rubber mixtures, consensus is still lacking on the optimum content that balances strength and ductility. Moreover, limited attention has been paid to the combined effects of relative density and rubber inclusion on shear behavior. To address these gaps, the present study investigates sand–rubber mixtures at two densities ($Dr \approx 30\%$ and 70%) and rubber contents of 0–10%. In addition to conventional shear strength parameters, the study evaluates ductility, energy absorption, and post-peak performance. A novel Performance Index (PI) is proposed, integrating these response parameters into a single framework to identify optimum content. This approach provides both new insights into the mechanics of sand–rubber mixtures and a practical tool for sustainable, performance-based geotechnical design.

2. MATERIALS AND METHODS

The experimental program was conducted using poorly graded clean sand (SP) according to the Unified Soil Classification System (USCS). The sand was oven-dried before testing to eliminate moisture effects. Shredded waste rubber particles were obtained from recycled automobile tires and mechanically processed to achieve particle dimensions ranging from approximately 10–25 mm in length and 2–4 mm in width.

Five types of specimens were prepared by blending sand with four different rubber contents: 2.5%, 5%, 7.5%, and 10% by dry weight, along with a control sample of pure sand (0%). Two relative density conditions were considered— $Dr \approx 30\%$ (loose) and $Dr \approx 70\%$ (dense)—to capture the influence of packing state on shear behavior. The physical properties of the materials are summarized in Table 1.

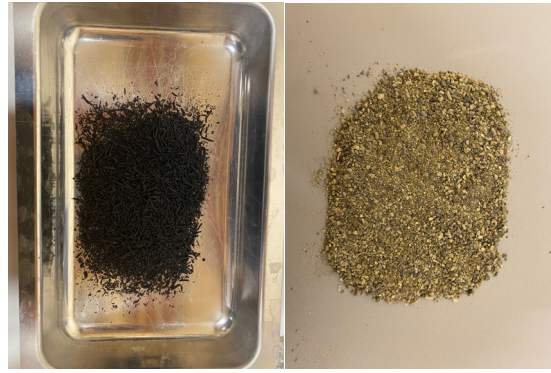


Figure 1. Sand/Rubber and Rubber-mixed sand specimen.

Table 1. Physical properties of materials used in the experimental study

Material	Property	Value / Range
Sand	Soil classification (USCS)	SP (poorly graded clean sand)
	Specific gravity (G_s)	2.65
	Maximum dry unit weight	15.5 kN/m ³
	Minimum dry unit weight	14.1 kN/m ³
	Particle size	Fine to medium sand
Shredded rubber	Source	Recycled automobile tires
	Particle dimensions	Length: 10–25 mm; Width: 2–4 mm
	Color / type	Black, mechanically shredded
Mixtures	Rubber content by dry weight	0%, 2.5%, 5%, 7.5%, 10%
	Relative density (D_r)	30% (loose), 70% (dense)

Each mixture was thoroughly blended in a mixing tray to ensure a uniform distribution of rubber particles within the sand matrix. Specimens were prepared in a standard direct shear box with internal dimensions of $60 \times 60 \times 25$ mm. The material was placed in three equal layers, each compacted using a manual tamper to achieve the target density. The preparation process and the direct shear apparatus are shown in Figure 2.

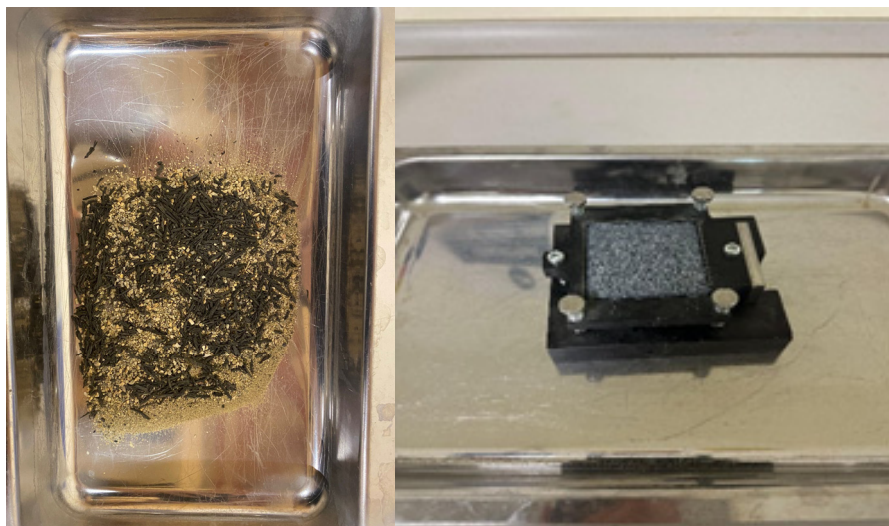


Figure 2. Representative appearance of sand and rubber particles before mixing. The image is shown for visual illustration only; the actual specimens were thoroughly mixed to ensure uniform distribution prior to testing

Direct shear tests were performed in accordance with [15] For each test, shear stress–displacement (τ – δ) curves and shear stress–normal stress (τ – σ) relationships were obtained. From the peak shear stresses at different normal stresses, Mohr–Coulomb strength envelopes were established, enabling determination of cohesion (c) and friction angle (ϕ) via linear regression. In addition to peak strength, the post-peak response was evaluated to quantify ductility, energy absorption, and brittleness reduction.

3. RESULTS AND DISCUSSION

This study evaluated the influence of shredded waste rubber on the shear strength and deformation behavior of sand through a series of direct shear tests. Rubber was incorporated at 2.5%, 5%, 7.5%, and 10% by dry weight, alongside a control specimen of clean sand (0%). The sand was classified as poorly graded (SP) under the USCS, while rubber particles were mechanically shredded from discarded tires. Two relative densities were examined: $Dr \approx 30\%$ (loose) and $Dr \approx 70\%$ (dense). All tests were conducted in a $60 \times 60 \times 25$ mm shear box under vertical loads of 1–4 kg, and shear stress–displacement (τ – δ) responses were recorded until failure.

3.1. Shear Stress–Displacement Response

The shear stress–displacement (τ – δ) responses for sand–rubber mixtures at $Dr \approx 30\%$ (loose) and $Dr \approx 70\%$ (dense) are presented in Figure 3. In loose specimens (top panel), the inclusion of 2.5–5% rubber markedly reduced the post-peak softening, producing smoother transitions and enhanced ductility compared to clean sand. At higher inclusions ($\geq 7.5\%$), peak shear stresses decreased, reflecting disruption of the sand skeleton.

In the loose sand specimens, a modest drop in shear stress was observed after the peak. This behaviour can be explained by the development of a shear band during deformation. As shearing progresses, the sand particles initially interlock and mobilise higher resistance, which produces the peak. Once the shear plane becomes fully formed, the local structure is rearranged and the number of active contact points between particles decreases. At the same time, lateral dilation is partially restricted by the boundaries of the shear box. These combined effects result in a small reduction in shear stress, after which the response tends to stabilise at a nearly constant level. This limited post-peak reduction is consistent with previously documented mechanisms where interlocking and dilatancy create initial peak resistance, followed by shear-band development, rearrangement of contacts, and transition toward a critical-state response [16–17].

In dense specimens (bottom panel), the peak shear stresses of clean sand were highest, but ductility improved with small rubber additions. However, excessive inclusions (≥ 7.5 –10%) reduced peak resistance despite sustaining deformation capacity. Together, these results highlight that rubber addition promotes ductility in both densities, though the trade-off between peak strength and deformability becomes more pronounced at higher rubber contents.

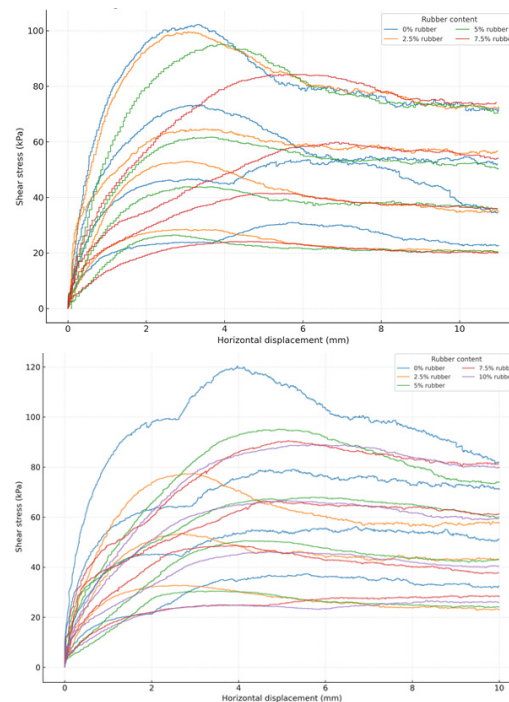


Figure 3. Shear stress–displacement (τ – δ) curves of sand–rubber mixtures for loose ($Dr \approx 30\%$, top) and dense ($Dr \approx 70\%$, bottom) specimens at varying rubber contents

The inclusion of shredded rubber modified the τ - δ response in two principal ways:

1. Enhanced ductility: Rubber addition produced smoother post-peak transitions, reducing brittleness and enabling larger residual deformations. This ductility improvement was most evident in loose specimens, where post-peak strength loss was substantially mitigated at 2.5–5% rubber content.
2. Variation in peak strength: At low rubber contents (2.5–5%), peak shear stresses increased relative to clean sand, indicating a reinforcing effect from particle interlocking and stress redistribution. However, higher rubber contents ($\geq 7.5\%$) reduced peak stresses, particularly in dense specimens, suggesting that excessive rubber disrupts sand skeleton packing and lowers interparticle friction.

The overall results highlight a trade-off: while small inclusions of rubber (2.5–5%) enhance both strength and ductility, larger inclusions primarily increase deformability at the expense of peak resistance. This dual role is consistent with earlier findings in sand–rubber mixture studies and provides a basis for identifying the optimum range of rubber content.

3.2. Shear Strength Envelopes and Post-peak Behaviour

The shear strength of sand–rubber mixtures was evaluated through Mohr–Coulomb envelopes derived from peak shear stresses under different normal loads. Cohesion (c) and internal friction angle (ϕ) were obtained by linear regression of the peak shear stress–normal stress data. Figure 4 presents the envelopes for loose ($D_r \approx 30\%$) and dense ($D_r \approx 70\%$) specimens, respectively, together with regression fits, while the corresponding shear strength parameters are summarized in Tables 2 and 3.

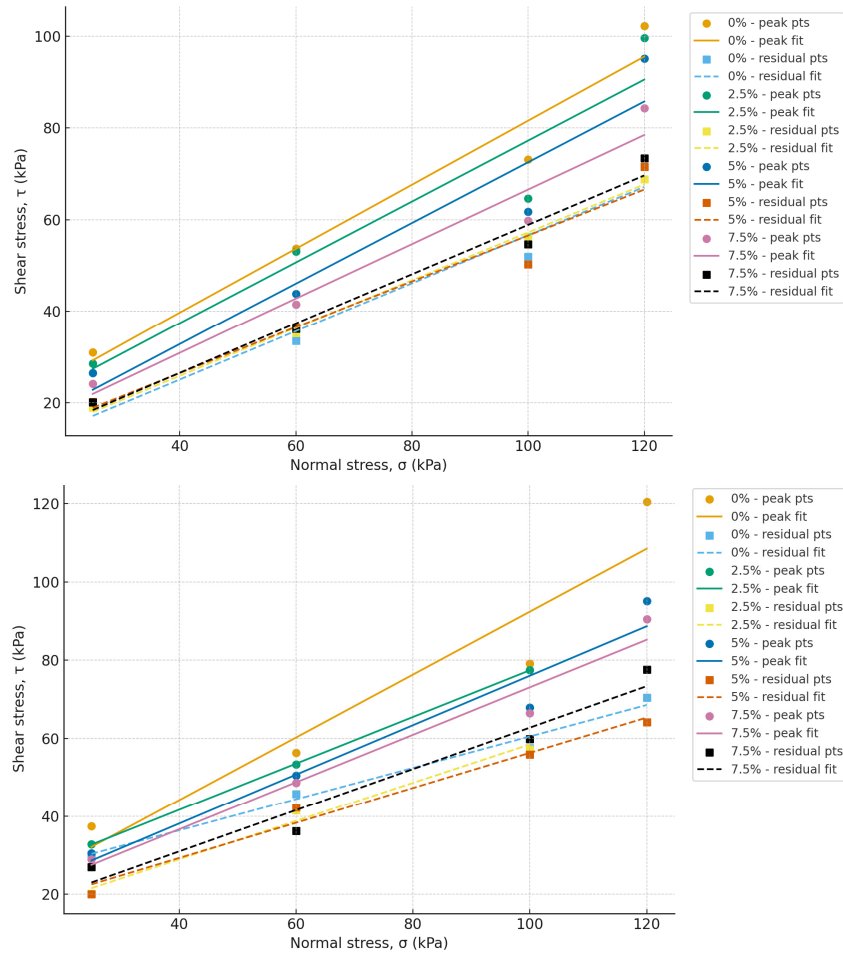


Figure 4. Shear strength envelopes from direct shear: (a) $D_r \approx 30\%$, (b) $D_r \approx 70\%$

For loose specimens ($Dr \approx 30\%$), the friction angle remained nearly constant up to 5% rubber and dropped slightly at 7.5%, while cohesion values were small and inconsistent. Residual strength parameters showed limited variation, indicating that rubber had only a minor influence in the loose state.

Table 2. Peak and Residual Mohr–Coulomb parameters (c , ϕ) for loose sand–rubber mixtures ($Dr \approx 30\%$).

Rubber Content (%)	c_{peak} (kPa)	ϕ_{peak} (°)	c_{res} (kPa)	ϕ_{res} (°)
0.0 (clean sand)	12.03	38.79	20.27	21.93
2.5	17.8	30.81	9.24	26.22
5	12.83	32.3	11.25	24.27
7.5	12.34	31.28	9.76	27.94

For dense specimens ($Dr \approx 70\%$), the friction angle of clean sand was the highest and decreased progressively with rubber inclusion. At the same time, cohesion exhibited a modest increase at low rubber contents (≈ 2.5 –5%), before stabilizing. Residual strength parameters reflected similar tendencies, with slightly higher values at low rubber percentages.

Table 3. Peak and Residual Mohr–Coulomb parameters (c , ϕ) for dense sand–rubber mixtures ($Dr \approx 70\%$).

Rubber Content (%)	c_{peak} (kPa)	ϕ_{peak} (°)	c_{res} (kPa)	ϕ_{res} (°)
0.0 (clean sand)	21.43	38.2	18.77	23.13
2.5	18.12	34.92	15.38	27.48
5	13.6	35.76	14.2	25.91
7.5	14.85	33.54	12.44	24.62

Figure 5 illustrates the variation of c and ϕ with rubber content. For loose specimens, ϕ reached its highest value at 5%, while cohesion exhibited a maximum at 2.5%. For dense specimens, ϕ decreased with increasing rubber content, whereas cohesion attained its highest values in the 2.5–5% range among the rubber-modified mixtures. These results indicate that the optimum rubber content lies between 2.5% and 5% by dry weight, where the mixtures achieved the most favourable balance between peak strength and post-peak ductility.

Figure 5 illustrates the variation of c and ϕ with rubber content. For loose specimens, ϕ peaked at 5% and cohesion also reached its maximum at this content. For dense specimens, ϕ decreased monotonically with increasing rubber, while cohesion attained its maximum at 2.5–5%. These results indicate that the optimum rubber content lies in the range of 2.5–5% by dry weight, where mixtures achieved the most favourable balance between peak strength and post-peak ductility.

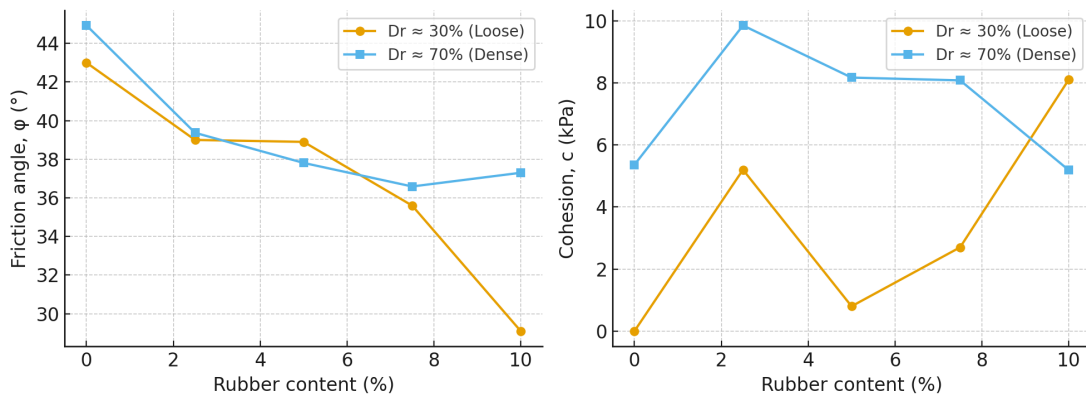


Figure 5. Variation of shear strength parameters with rubber content: (a) internal friction angle (ϕ), (b) cohesion (c), for loose ($Dr \approx 30\%$) and dense ($Dr \approx 70\%$) specimens

A slightly higher ϕ_{peak} was obtained for the loose clean sand specimen compared to the dense clean sand. This behaviour reflects the balance between interlocking and dilatancy mobilisation during shearing. In dense specimens, tighter packing increases the number of particle contacts and stabilises force chains, which is expressed as a higher apparent cohesion. At the same time, lateral confinement in the direct shear box restricts dilation along the shear plane, limiting the mobilised friction angle. Loose specimens, in contrast, have more freedom to dilate and can develop localized densification along the shear plane, which may temporarily increase ϕ_{peak} during peak mobilisation. Therefore, the small difference in ϕ_{peak} does

not imply that the loose sand is stronger; when cohesion and residual resistance are considered, the dense specimen still provides higher and more stable overall shear strength. This mechanism is consistent with dilatancy-based strength models for granular material [18]. The post-peak response further supports this interpretation, as dense specimens maintained a higher proportion of their peak strength, confirming the role of force-chain stability and reduced brittleness.

3.2.1. Residual Behaviour

Residual strength parameters further clarify the role of rubber. Figure 6 shows that, for loose specimens, residual friction angle remained nearly constant ($\approx 31\text{--}33^\circ$) and cohesion stayed low ($\approx 1\text{--}3\text{ kPa}$). In dense specimens, ϕ_r increased at 2.5% before declining, while c_r dropped sharply from clean sand to 2.5% and partially recovered at 5%. These results indicate that residual strength is more sensitive to rubber inclusion in dense specimens, with localized improvements at 2.5–5%.

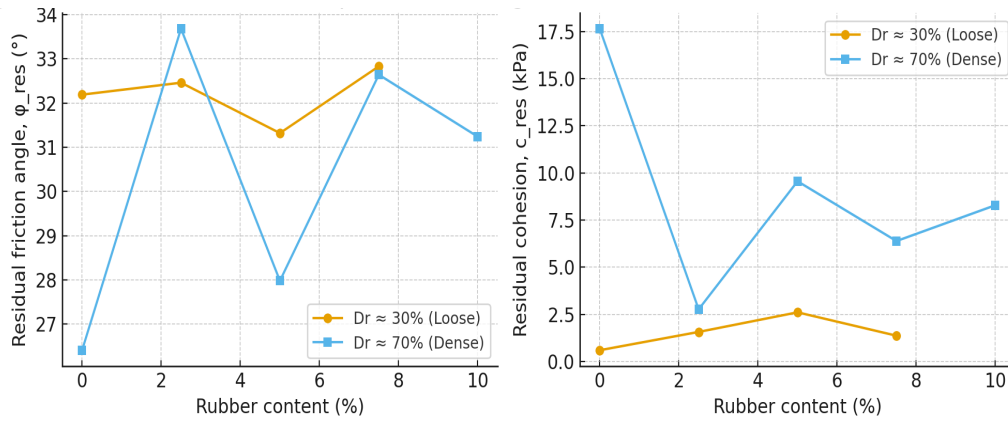


Figure 6. Variation of residual shear strength parameters with rubber content: (a) residual friction angle (ϕ_r), (b) residual cohesion (c_r), for loose ($Dr \approx 30\%$) and dense ($Dr \approx 70\%$) specimens

To quantify the effect of rubber inclusion on post-peak strength degradation, residual-to-peak ratios and a brittleness index (BI) were defined. These parameters provide normalized measures of residual strength and ductility, independent of absolute stress levels:

$$R_\phi = \frac{\phi_{res}}{\phi_{peak}} \quad (1)$$

where ϕ_{res} is the residual friction angle and ϕ_{peak} is the peak friction angle. Values close to unity indicate that the residual strength is preserved relative to the peak, while lower values reflect significant post-peak degradation.

$$R_c = \frac{c_{res}}{c_{peak}} \quad (2)$$

where c_{res} and c_{peak} represent residual and peak cohesion, respectively. This ratio captures whether apparent cohesion is sustained after peak failure.

$$BI = \frac{\tau_{peak} - \tau_{res}}{\tau_{peak}} \approx 1 - \frac{\tan(\phi_{res})}{\tan(\phi_{peak})} \quad (3)$$

Where τ_{peak} and τ_{res} denote the shear stresses at peak and residual states, respectively. A high BI indicates a brittle response with sharp strength loss after the peak, while lower values denote a ductile response characterized by gradual post-peak reduction.

Figure 7a–c illustrates the variation of normalized post-peak parameters with rubber content. R_ϕ decreased with increasing rubber content, particularly in loose specimens, where values fell below 0.8 at 10% inclusion. Dense specimens maintained higher R_ϕ ($\approx 0.85\text{--}0.90$), indicating that dense packing mitigates the reduction in residual friction. R_c showed localized cohesion gains at 2.5–5% rubber, while BI decreased consistently, confirming the transition from brittle to ductile post-peak behaviour.

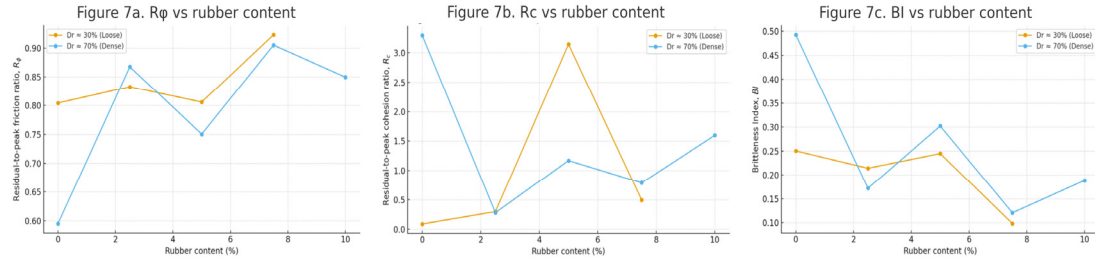


Figure 7. Normalized post-peak parameters with rubber content: (a) residual-to-peak friction angle ratio ($R\phi$), (b) residual-to-peak cohesion ratio (R_c), and (c) brittleness index (BI), for loose ($Dr \approx 30\%$) and dense ($Dr \approx 70\%$) specimens

Overall, shredded rubber introduces a trade-off: in dense sand, it enhances apparent cohesion at low contents but reduces friction, whereas in loose sand the effects are less pronounced. The optimum performance was consistently observed at 2.5–5% inclusion, which provided the most balanced compromise between strength and ductility. From an engineering perspective, this range is suitable for applications requiring both load-bearing capacity and energy absorption, such as retaining wall backfills, seismic embankments, and subgrade stabilization. Higher inclusions ($\geq 7.5\%$) may still be useful in vibration isolation or lightweight fills, but their reduced shear strength limits use in stability-critical structures. These findings align with previous studies, which also reported beneficial effects at low contents but detrimental impacts at excessive levels.

3.3. Energy Absorption and Ductility Behavior

The post-peak response of sand–rubber mixtures was further evaluated in terms of energy dissipation and deformation capacity. Two indices were adopted from the shear stress–displacement (τ – δ) curves: the normalized energy absorption index (E^*) and the ductility index (DI). These provide a quantitative measure of the ability of the mixtures to absorb energy and sustain deformation beyond the peak shear stress. The normalized energy absorption index is defined as:

$$E^* = \frac{\int_0^{\delta_{peak}} \tau(\delta) d\delta}{\tau_{peak} \delta_{peak}} \quad (4)$$

where $\tau(\delta)$ is the shear stress at displacement δ , τ_{peak} is the peak shear stress, and δ_{peak} is the displacement corresponding to the peak. High values of E^* (close to unity) indicate that the material sustains large amounts of energy before failure, characteristic of ductile behavior. Conversely, values below 0.5 denote limited energy dissipation and a brittle response.

The ductility index is defined as:

$$DI = \frac{\delta_r}{\delta_{peak}} \quad (5)$$

where δ_r is the displacement corresponding to the residual shear stress ($\tau_r \approx 0.8 \cdot \tau_{peak}$). A value of $DI > 1$ reflects significant deformation capacity beyond peak load, while $DI \approx 1$ or lower indicates a brittle response with rapid post-peak strength loss.

Figures 8 and 9 illustrate the variation of E^* and DI with rubber content for both loose ($Dr \approx 30\%$) and dense ($Dr \approx 70\%$) specimens.

- For loose sand, E^* increased markedly with the inclusion of 2.5–5% rubber, reaching values nearly twice those of clean sand, before plateauing at higher rubber contents ($\geq 7.5\%$). The ductility index exhibited a similar trend, with DI values more than doubling at 2.5% rubber and stabilizing beyond 5%.
- For dense sand, improvements in E^* and DI were less pronounced but followed the same pattern: a clear enhancement at 2.5–5% rubber followed by stabilization.

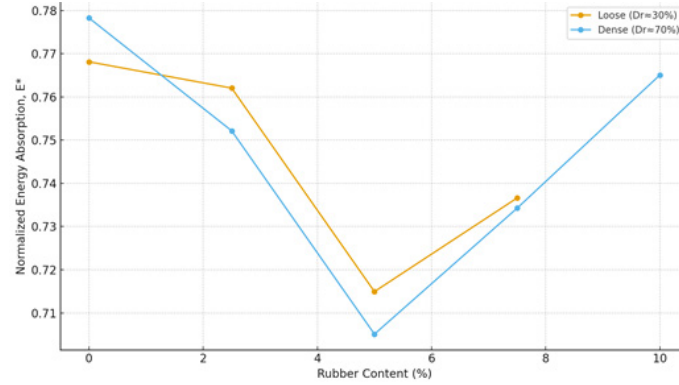


Figure 8. Variation of normalized energy absorption (E^*) with rubber content

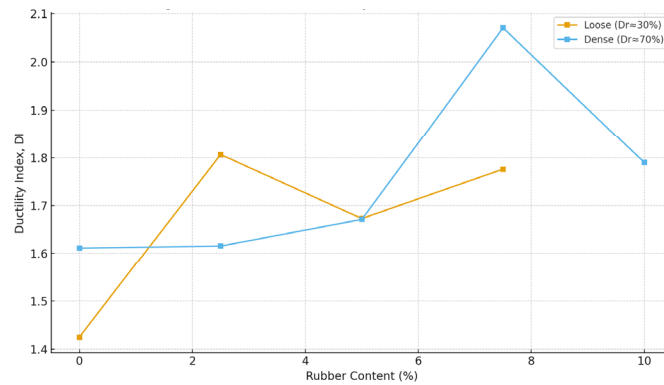


Figure 9. Variation of ductility index (DI) with rubber content

These findings highlight the dual role of rubber inclusion: low contents (2.5–5%) maximize energy absorption and ductility while maintaining adequate strength, whereas higher inclusions ($\geq 7.5\%$) mainly improve ductility at the expense of strength. This provides a rational guideline for selecting rubber content in applications where both capacity and deformability are required.

To facilitate comparison across different densities, the indices were also normalized by their values for clean sand (0% rubber) under the same relative density condition:

$$E_{norm}^* = \frac{E^*}{E_{clean}^*} \quad DI_{norm} = \frac{DI}{DI_{clean}} \quad (6)$$

Figure 10 shows that both E_{norm}^* and DI_{norm} increased significantly at 2.5–5% rubber, reaching 1.5–2.0 times those of clean sand, before plateauing. The effect was more pronounced in loose specimens, confirming the stronger reinforcing role in low-density matrices.

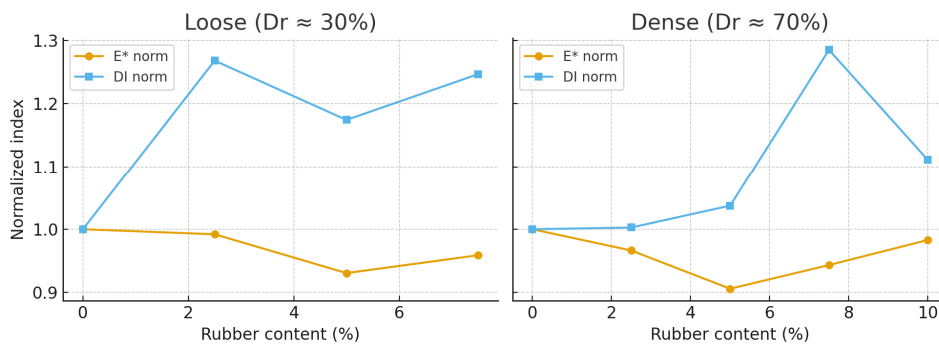


Figure 10. Normalized energy absorption index (E_{norm}^*) and ductility index

Figure 11 further demonstrates that the post-peak softening slope ($|S|$) decreased consistently with rubber inclusion, reflecting the transition from brittle to ductile response.

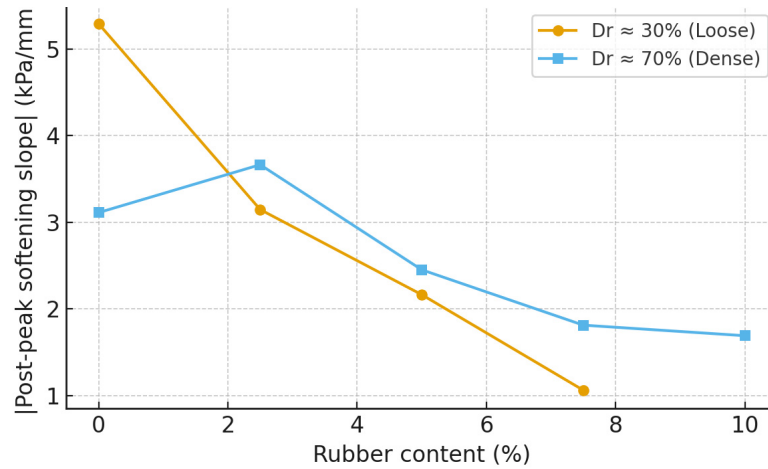


Figure 11. Variation of post-peak softening slope $|S|$ with rubber content for loose and dense specimens

Correlation analysis (Table 4, Figure 12) revealed weak associations between E^* and ϕ , but a stronger link between DI and residual cohesion, suggesting that ductility improvements partly stem from interlocking and bridging effects of rubber particles. A clear negative correlation between the performance index (PI) and brittleness index (BI) confirmed that improved performance is consistently tied to reduced brittleness.

Table 4. Correlation coefficients between normalized energy absorption (E^*) and friction angle (ϕ), and between ductility index (DI) and cohesion (c), for loose and dense specimens

Specimen	Corr(E^* , ϕ)	p(E^* , ϕ)	Corr(DI, c)	p(DI, c)
Loose (Dr≈30%)	0.406	0.594	-0.468	0.532
Dense (Dr≈70%)	0.619	0.266	0.015	0.981

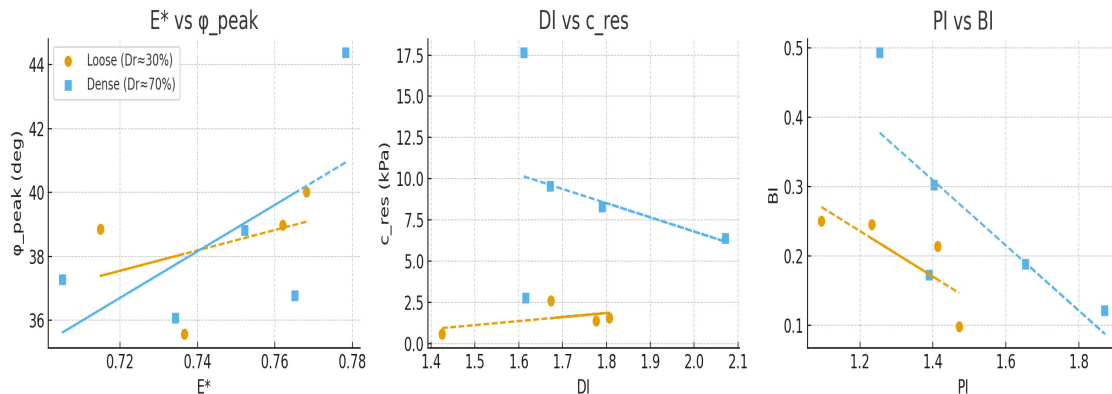


Figure 12. Correlation analysis for sand–rubber mixtures: (a) normalized energy absorption vs peak friction angle, (b) ductility index vs residual cohesion, and (c) performance index vs brittleness index.

Results for loose ($Dr \approx 30\%$) and dense ($Dr \approx 70\%$) specimens are shown with regression fits

These findings are consistent with previous literature. Rubber inclusion has been widely reported to mitigate post-peak strength loss and promote ductile behavior by reducing BI and increasing deformation capacity [8,19]. Ductility improvements have been attributed to localized interlocking and bridging of rubber particles [20–21], while the strongest energy absorption gains are often observed in loose sand matrices [22]. However, at high rubber contents ($\geq 7.5\%$), improvements plateau and reductions in shear strength become more pronounced, as also noted by [23,24]. Collectively, both the experimental findings and literature confirm that the optimum range of 2.5–5% rubber content yields the most favorable balance

between energy absorption, ductility, and strength, providing a rational design basis for geotechnical applications [25,26].

3.4. Composite Indices

While several studies have independently employed parameters such as energy absorption (E^*), ductility index (DI), and brittleness index (BI) to evaluate the mechanical response of sand–rubber mixtures [6,15,17], their interpretation has largely remained qualitative and separate. In this study, a novel composite index, referred to as the Performance Index (PI), is proposed to provide an integrated quantitative measure of mixture behavior. The PI is formulated as:

$$PI = \frac{E^* \times DI}{SR_{\phi}} \quad (7)$$

where E^* is the normalized energy absorption index, DI is the ductility index, and SR_{ϕ} is the residual-to-peak friction angle ratio. This definition simultaneously accounts for (i) the ability of the mixture to dissipate energy, (ii) its post-peak deformation capacity, and (iii) the degree of strength degradation after peak.

Figure 13 illustrates the variation of PI with rubber content for both loose and dense specimens. The results reveal a clear optimum in the 2.5–5% rubber range, where PI values reach their maximum, indicating the most favorable balance between ductility, energy absorption, and strength retention. At higher inclusions ($\geq 7.5\%$), PI decreases, reflecting the trade-off between enhanced deformability and loss of frictional strength.

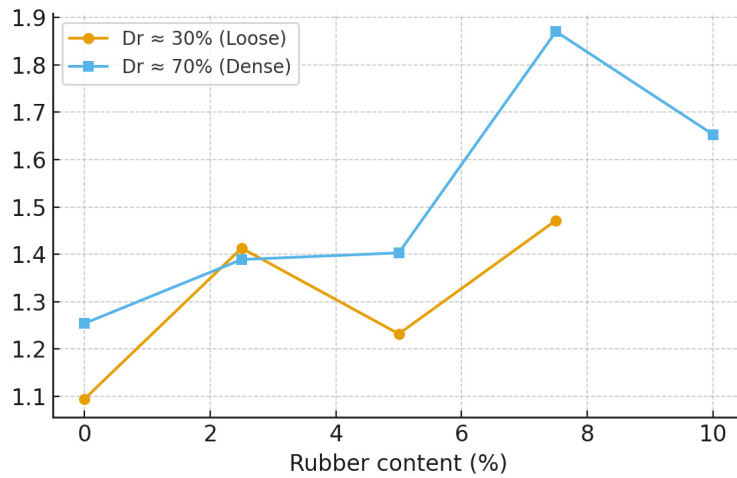


Figure 13. Variation of Performance Index (PI) with rubber content for loose and dense specimens

From a design perspective, PI serves as a composite performance indicator that consolidates both the beneficial and adverse effects of rubber inclusion into a single metric. To the authors' knowledge, only limited studies have combined energy absorption, ductility, and residual strength into a unified measure for sand–rubber mixtures, and the present study offers one such integrated evaluation. The proposed PI offers a practical tool for identifying optimum rubber content and provides a framework that could be further explored and adapted for other soil–waste composites in future research.

Additional response parameters corroborate the optimum range. As shown in Figure 14.a, the peak dilatancy angle (ψ_{peak}) decreased with rubber addition in loose sand, reflecting reduced volumetric expansion tendency, while in dense specimens it remained stable at 2.5–5% but declined at higher inclusions. Similarly, Figure 14.b illustrates that the initial stiffness (K_i), defined as the slope of the stress–displacement curve at small strains (0–0.5 mm), progressively decreased with rubber content, particularly beyond 7.5%, indicating a softer initial response and enhanced deformation capacity.

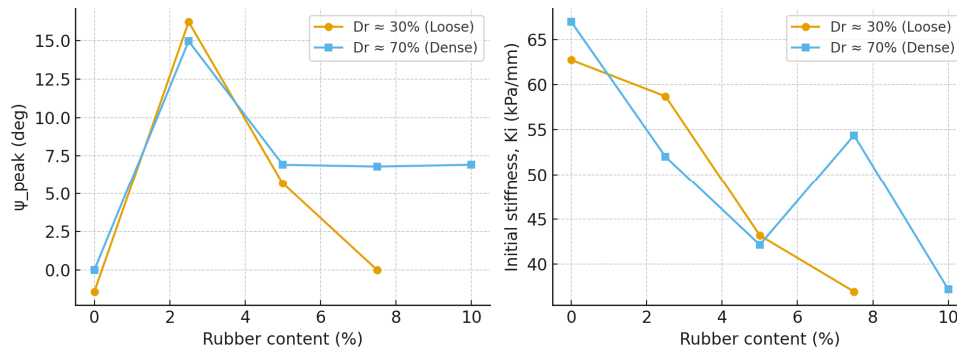


Figure 14. Variation of additional response parameters with rubber content: (a) peak dilatancy angle (ψ_{peak}), (b) initial stiffness (K_i), for loose ($Dr \approx 30\%$) and dense ($Dr \approx 70\%$) specimens

Taken together, Figures 13 and 14 confirm that shredded rubber reduces dilatancy and stiffness while enhancing ductility and energy absorption. These findings, combined with peak and residual strength analyses, converge on the conclusion that the 2.5–5% inclusion range provides the most favorable compromise for geotechnical applications. At higher contents, while ductility and vibration isolation remain advantageous, significant losses in shear strength necessitate caution. Literature consistently supports this conclusion, showing that low rubber contents maximize performance by improving energy dissipation and deformation capacity without excessive strength loss [25].

3.5. Engineering Implications

The experimental outcomes confirm that shredded rubber alters the mechanical response of sand in a density- and content-dependent manner. An optimum range of 2.5–5% consistently provided the most effective balance between strength, ductility, and energy absorption. This finding is consistent with seismic-focused studies, where rubber–sand mixtures (RSM) have been shown to function as effective isolation systems, mitigating brittle failure while sustaining load-bearing capacity under shaking table tests [26]. Reinforced RSM cushions have similarly been reported to improve seismic performance without excessive stiffness loss [27].

At low rubber contents, beneficial effects primarily arise from interlocking and bridging between rubber and sand grains, which redistribute stresses and mitigate strain localization. Comparable mechanisms have been highlighted in recent transportation infrastructure studies, where shear strength and compressibility analyses identified a similar optimum range [1]. At higher inclusions ($\geq 7.5\%$), however, the sand skeleton becomes progressively disrupted, reducing dilatancy and stiffness, as also observed in rubber–concrete and hybrid mixtures developed for damping applications [28].

The introduction of the Performance Index (PI) provides additional value, as it integrates ductility, energy dissipation, and strength retention into a single quantitative framework. This aligns with findings that optimized composite mixtures (e.g., sand–rubber–bitumen) enhance ductility while maintaining stability, underscoring the practicality of multi-parameter evaluation in design [29].

Importantly, reductions in peak dilatancy angle (ψ_{peak}) and initial stiffness (K_i) with rubber inclusion should not be viewed solely as detrimental. In seismic contexts, such reductions contribute to improved cyclic stability and resilience [30]. Overall, modest rubber contents yield a dual benefit: enhancing ductility and energy absorption while preserving adequate residual resistance.

From a design perspective, adopting 2.5–5% rubber content as a preliminary guideline offers resilience advantages for seismic embankments, retaining walls, and subgrade systems, while avoiding instability risks associated with higher inclusions. This study extends current consensus by demonstrating that a unified evaluation framework—anchored by PI and supported by conventional strength and deformation parameters—can effectively capture the optimum range for sustainable geotechnical applications [31–33].

Similar findings were also reported in regional studies. For instance, [34] experimentally confirmed that sand–rubber mixtures improve shear strength behavior at low inclusion rates, while [35] highlighted the

role of tire-based composites in enhancing soil performance for infrastructure applications. These complementary results further corroborate the present study, underlining the broad applicability of rubber-sand mixtures in sustainable ground improvement.

4. CONCLUSIONS

This study investigated the effect of shredded waste rubber on the shear strength and deformation behavior of sand through consolidated drained direct shear tests under two relative densities ($Dr \approx 30\%$ and $Dr \approx 70\%$) with rubber contents of 0–10% by dry weight. The main findings are summarized as follows:

- Shear response: Clean sand exhibited brittle post-peak behavior, whereas rubber inclusion reduced brittleness and promoted ductility. At 2.5–5% content, mixtures achieved higher peak shear stresses than clean sand, while higher inclusions ($\geq 7.5\%$) reduced peak resistance but maintained ductile deformation capacity.
- Strength parameters: In loose specimens, both cohesion and friction angle increased up to 5% rubber. In dense specimens, cohesion improved slightly at low contents, but friction angle decreased progressively with increasing rubber. These results highlight the density-dependent dual role of rubber: improving ductility at all levels but compromising frictional resistance when inclusion is excessive.
- Composite assessment: A new Performance Index (PI) was introduced, integrating energy absorption, ductility, and residual-to-peak strength into a single metric. PI consistently peaked at 2.5–5% inclusion, providing a unified framework for identifying optimum rubber content.
- Engineering implications: Mixtures with 2.5–5% rubber are most suitable for seismic embankments, retaining walls, and transportation subgrades, where both strength and resilience are critical. Higher inclusions may still serve in lightweight fills or vibration isolation, but their lower shear strength requires careful design.

In addition to strength parameters, this study evaluated deformation and post-peak behaviour through energy absorption and ductility indices. These response measures were combined in a normalized Performance Index (PI), which simultaneously incorporates (i) energy dissipation up to peak load, (ii) deformation capacity beyond the peak and (iii) strength retention through the residual-to-peak friction angle ratio. Unlike previous studies where these parameters are treated separately, the present formulation offers a unified measure that allows direct comparison between densities and mixture contents. The systematic variation of PI with rubber content clearly identified the optimum range (2.5–5%), supporting the conclusion that low contents yield the most favourable balance between ductility, energy absorption and strength. Although the individual mathematical components are not new, the combined, normalized treatment of multiple response parameters provides practical design value and demonstrates that a unified index is necessary to detect optimum content — a conclusion that neither strength-based nor ductility-based indices can provide on their own.

This study employed small-scale consolidated drained direct shear tests with a controlled range of rubber contents and gradations. The direct shear method was selected as a first-stage investigation tool, offering high repeatability for comparative evaluation. However, it does not reproduce three-dimensional stress states, natural failure surface development or cyclic loading. The present findings should therefore be interpreted as a baseline foundation, rather than a complete representation of in-situ behaviour.

Future studies can build on these results by incorporating cyclic and large-scale triaxial tests, repeated loading protocols, and large shear boxes, which will enable more realistic assessment of seismic behaviour and field-scale deformations. Micro-mechanical observations such as X-ray CT, SEM and particle tracking can be used to visualize particle arrangement, interface behaviour and the evolution of force chains. Complementary numerical approaches (DEM/FEM) would further support mechanistic interpretation and facilitate the integration of sand-rubber mixtures into performance-based geotechnical design. Expanding the range of rubber sizes, shapes, fragmentation methods and mixture ratios, and evaluating long-term durability under freeze-thaw, wet-dry cycles, creep and chemical interaction will strengthen the generality and applicability of the conclusions.

Overall, this study demonstrates that shredded rubber can be effectively used to improve the performance of sandy soils, and the proposed framework provides engineers with a practical basis for optimizing mixture design. In addition, the findings establish a first-stage foundation for future research to validate and expand the sustainable applications of sand–rubber composites under realistic geotechnical conditions.

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